## Physics Laboratory practical Total marks [12.6]

## Title: Determination of coefficient of viscosity of oil

## Determination of coefficient of viscosity of fluids

## Ball Drop Experiment

The measurement involves determining the velocity of a falling sphere through a column of fluid of unknown viscosity. This is accomplished by dropping a sphere through a measured distance of fluid and measuring how long it takes to traverse the distance.

## Materials

- Thermometer
- Ball bearings of different diameters
- Stopwatch
- Meter ruler
- Paper towel
- Magnet
- Oil


## Theoretical aspects

Consider a spherical ball bearing of radius $r$ and density $\rho_{s}$ falling through a column of viscous fluid of coefficient of viscosity $\eta$ and density $\rho_{f}$ as
illustrated in the figure below. The coefficient of viscosity is a measure of the degree of internal resistance to flow and shear.


Figure 1: showing a sphere of radius $r$ falling through a column of fluid of density $\rho_{f}$. A and B marks the distance travelled by the sphere at terminal velocity $v_{t}$.

According to Newton's second law:

$$
\begin{gather*}
\text { Net Force }=m a \\
m a=W-\left(F_{u}+F_{v}\right) \tag{1}
\end{gather*}
$$

Where $m$ is the mass of the sphere,
$W=m g$, is the weight of the sphere (ball bearing)
$F_{u}=\frac{4}{3} \pi r^{3} \rho_{f} g$ is the upthrust $=$ weight of the fluid displaced
$F_{v}=6 \pi r \eta v$ is the viscous force (of a sphere of radius $\eta$ ) proportional to the velocity $v$ of the ball (Stoke's Law).

Initially the ball has some downward acceleration until the sphere acquires terminal velocity $v_{t}\left(v_{t}=\frac{s}{t}\right.$ where $\boldsymbol{s}$ is the distance travelled in time $\boldsymbol{t}$, when there is no more acceleration and hence the net force is zero. Equation (1) becomes

$$
\begin{gather*}
m g=F_{u}+F_{v} \\
\frac{4}{3} \pi r^{3} \rho_{s} g=\frac{4}{3} \pi r^{3} \rho_{f} g+6 \pi r \eta v_{t}  \tag{2}\\
\text { Or } v_{t}=\frac{2}{9} \frac{r^{2}}{\eta} g\left(\rho_{s}-\rho_{f}\right) \tag{3}
\end{gather*}
$$

Note that $\boldsymbol{s}$ is the distance between A and B and $\boldsymbol{t}$ is the time the ball takes to fall between $A$ and $B$.

Equation (3) can be modified to:

$$
\begin{equation*}
v_{t}=\frac{1}{18} \frac{d^{2}}{\eta} g\left(\rho_{s}-\rho_{f}\right) \tag{4}
\end{equation*}
$$

Where
$\mathrm{d}=$ diameter of sphere ( $=2 \mathrm{r}$ )
$\rho_{s}=$ density of sphere $=\mathrm{m} / \mathrm{V}=$ (mass of sphere/volume of sphere)
$\rho_{f}=$ density of fluid
$\mathrm{g}=$ acceleration of gravity $=9.81 \mathrm{~m} / \mathrm{s}^{2}$
$v_{t}=$ Terminal Velocity $=\mathrm{s} / \mathrm{t}=($ distance sphere falls $) /($ time of it takes to fall)

## Procedure

Proceed as follows.

1. Measure the vertical distance $\boldsymbol{s}$ between points $\mathbf{A}$ and B marked on the cylindrical tubes.
2. Drop one of the ball bearings into the fluid (ensuring that the ball bearing does not touch the wall of the cylinder during its motion between $A$ and $B$ )
3. Measure the time $t$ taken by the sphere to travel the distance $s$ between A and B and record it in the provided table.
4. Without removing the ball bearing, repeat steps 2 and 3 above using other bearings of the same diameter to have three values of time.
5. Repeat steps 2 to 4 for the other 4 sizes of ball bearings.

## Results and analysis

Note the following:

```
\(\rho_{f}=871.4 \mathrm{~kg} / \mathrm{m}^{3}\)
\(\rho_{s}=7717 \mathrm{~kg} / \mathrm{m}^{3}\)
\(s=0.4 \mathrm{~m} \quad\) (distance between A and B )
```

Calculate the average time, $d^{2}$ and $v_{t}$ for each set of ball bearings, complete Table 1.

Table 1: experimental results.
[2.4]

| Ball diameter |  | Diameter <br> squared | Time taken to fall distance $\boldsymbol{l}$ |  |  |  | Terminal <br> velocity |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\#$ | $\boldsymbol{d}(\mathrm{~mm})$ | $\boldsymbol{d}(\mathrm{m})$ | $\boldsymbol{d}^{2}\left(\mathrm{~m}^{2}\right)$ | $\boldsymbol{t}_{\mathbf{1}}(\mathrm{s})$ | $\boldsymbol{t}_{\mathbf{2}}(\mathrm{s})$ | $\boldsymbol{t}_{\mathbf{3}}(\mathrm{s})$ | Average <br> time $(\mathrm{s})$ | $\boldsymbol{v}_{\boldsymbol{t}}(\mathrm{m} / \mathrm{s})$ |
| 1 |  |  |  |  |  |  |  |  |
| 2 |  |  |  |  |  |  |  |  |
| 3 |  |  |  |  |  |  |  |  |
| 4 |  |  |  |  |  |  |  |  |

1. Plot a graph of $v_{t}$ vs $d^{2}$, [5.2]
2. Use the graph to determine the viscosity of the oil with the appropriate units.

## MARKING SCHEME Solution

## Completed table

Note the following:
$\rho_{f}=871.4 \mathrm{~kg} / \mathrm{m}^{3}$
$\rho_{s}=7717 \mathrm{~kg} / \mathrm{m}^{3}$

Temperature before $T_{b}$ :
$\boldsymbol{T}=\quad$ [0.15]
Distance $l$
$l \approx 0.500 \mathrm{~m}$
[0.25]
Drawing a sketchof measurement
[0.75]
Temperature before $T_{a}$ :
$\boldsymbol{T}_{\boldsymbol{a}}=$
[0.15]

## Points for the measurements and calculations

Table III-1

| \# | $\frac{\boldsymbol{d}}{(\mathrm{mm})}$ | d <br> (m) | $\begin{aligned} & \boldsymbol{d}^{2} \\ & \quad\left(\mathrm{~m}^{2}\right) \end{aligned}$ | $\boldsymbol{t}_{1}$ <br> (s) | $\boldsymbol{t}_{2}$ <br> (s) | $\underline{\boldsymbol{t}}_{3}$ | Average time (s) | $v_{t}(\mathrm{~m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0,25 | 0,05 | 0,05 | 0,25 | 0,25 | 0,25 | 0,05 | 0,1 |
| 2 | 0,25 | 0,05 | 0,05 | 0,25 | 0,25 | 0,25 | 0,05 | 0,1 |
| 3 | 0,25 | 0,05 | 0,05 | 0,25 | 0,25 | 0,25 | 0,05 | 0,1 |
| 4 | 0,25 | 0,05 | 0,05 | 0,25 | 0,25 | 0,25 | 0,05 | 0,1 |

## Subtraction (per column) 0.2 if out of range

| $\mathbf{d}(\mathbf{m m})$ | Range d <br> $(\mathrm{mm})$ | $\mathbf{d}_{\text {min }}(\mathbf{m m})$ | $\mathbf{d}_{\max }(\mathbf{m m})$ |
| :---: | :---: | :---: | :---: |
| $\mathbf{4 , 5 0}$ | 0,4 | 4,30 | $\mathbf{4 , 7 0}$ |
| $\mathbf{5 , 0 0}$ | 0,4 | 4,80 | 5,20 |
| $\mathbf{6 , 0 0}$ | 0,4 | 5,80 | $\mathbf{6 , 2 0}$ |
| $\mathbf{7 , 0 0}$ | 0,4 | $\mathbf{6 , 8 0}$ | $\mathbf{7 , 2 0}$ |

III-2 Plot of $\boldsymbol{v}_{\boldsymbol{t}}$ vs. $\boldsymbol{d}^{\mathbf{2}}$
Marks allocation:
i) $\quad x$ and $y$ axis labelling
(for each axis Quantity (0.25) \& Unit (0.25))
ii) Scale of the graph
0.5 for each axis (uniform \& size)
iii) Plotting of points ( 0.1 for each)
iv) Drawing straight line best of fit

Title: coefficeint of viscosity determination

III-3 Determining the slope of the line ..... [1.5]
i. Mark the points on the line that are used ..... [0.5]
ii. Calculation of the slope ..... [0.5]
iii. Determining the correct unit ..... [0.5]
III-4 Derive and show analytical expression for $C$ ..... [1.0]
i.

## III-5 Determination of viscosity of oil

Mark allocation
i) Determination of coefficient of viscosity.

From

$$
\begin{align*}
& v_{t}=\frac{1}{18} \frac{d^{2}}{\eta} g\left(\rho_{s}-\rho_{f}\right) \\
& \text { slope }=\frac{g}{18} \frac{\left(\rho_{s}-\rho_{f}\right)}{\eta} \quad \text { or } \quad \text { slope }=\frac{C}{\eta} \tag{0.5}
\end{align*}
$$

Calculation of the viscosity
Determining the correct unit

$$
\eta=\frac{g}{18} \frac{\left(\rho_{s}-\rho_{f}\right)}{\text { slope }} \rightarrow \frac{\left[\frac{m}{s^{2}}\right] *\left[\frac{k g}{m^{3}}\right]}{\left[\frac{m}{m^{2}}\right]}=\left[\frac{N s}{m^{2}}\right]=\text { Pa.S }
$$

